

<b>Course Title:</b> Partial Differential Equations	<b>Number of Units:</b> 1
<b>SSD :</b> MAT05	<b>CFU:</b> 6
<p><b>Course aims:</b> The course aims to provide basic knowledge of Partial Differential Equations, both needed to formulate mathematical models of engineering and scientific problems.</p>	
<p><b>Course Description:</b> Laplace Equation: physical interpretation and probabilistic interpretation. Fundamental solution, Newtonian potential. Harmonic functions: mean value formula, maximum principle, Harnack inequality, Liouville theorem. Uniqueness of solutions of Dirichlet and Neumann problems for the Poisson equation. Regularity of harmonic functions. Green function. The Green function in a half space and the Green function in a ball. Variational formulation of Dirichlet problem for the Poisson equation.</p> <p>Heat Equation: physical and probabilistic interpretation. Existence of global solutions. Solution of nonhomogeneous problems via Duhamel principle. Mean value properties of solutions and maximum principle. Uniqueness of solutions. Infinite speed propagation of the data. Energy methods. Backward uniqueness.</p> <p>Transport equation: solutions of the homogeneous and nonhomogeneous problem.</p> <p>Wave equation: physical interpretation. Solutions of the 1-dimensional homogeneous wave equation: D'Alembert formula. Solutions of the 2 and 3-dimensional homogeneous wave equation. Solutions of the nonhomogeneous wave equation via Duhamel principle. Characteristic cone and finite speed propagation of the data. Energy methods and uniqueness of solutions.</p> <p>Separation of variables: solutions of the Poisson equation in special planar and 3-dimensional domains, solutions of the heat equation and of the porous media equation.</p> <p>Fourier transform and applications to Bessel potentials and solutions of the heat equation, the Schrödinger equation, the wave equation and the telegraph equation.</p> <p>Laplace transform and applications to the heat equation and the wave equation. Compact operators. Fredholm theory. Eigenvalues and spectrum of a compact operator. Eigenvalues of the Laplacian: variational definition and properties. Properties of eigenfunctions.</p> <p>Sobolev spaces: definitions, basic properties, <math>H=W</math>, approximation with smooth functions, extension domains, traces, imbedding theorems and compact imbeddings, Poincaré inequality.</p> <p>Weak solutions of elliptic equations. Existence of weak solutions. Existence of strong and classical solutions via regularity.</p>	
<p><b>Assumed Background:</b> Mathematical Analysis at undergraduate level, basic knowledge of Real and Functional Analysis.</p>	
<p><b>Assessment methods:</b> Oral examination</p>	