Module Announcement

PhD in Information Technology and Electrical Engineering

Università degli Studi di Napoli Federico II

Module Title: Delay differential equations (DDEs) and their applications

Lecturer: Prof John Hogan

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CV: Prof John Hogan has been Professor of Mathematics in the University of Bristol since 1992. His work on piecewise smooth systems and delays has led to major breakthroughs in the understanding of systems involving impacts, friction and switches. He was the only UK Applied Mathematician to give a Plenary talk at the 7th International Congress on Industrial & Applied Mathematics (ICIAM 2011), Vancouver, British Columbia, Canada. He was awarded an Honorary Degree by the Faculty of Mechanical Engineering, Budapest University of Technology & Economics, Hungary in 2010. He leads the Bristol Centre for Applied Nonlinear Mathematics. He has been PI on EPSRC grants worth over £12M.

Dates and Locations (rooms are in bldg. 3/A, via Claudio 21, Napoli)

Date	Hours	Room
2 July 2018	**10:00-12:00**	**I6**
3 July 2018	**14:00-16:00**	**II2**
4 July 2018	**14:00-16:00**	**I5**
5 July 2018	**14:00-16:00**	**T5**
6 July 2018	**14:00-16:00**	**T5**

Content

Modeling mechanical or robotic systems usually involves control since feedback is used in order to maintain a stable state. This feedback requires a finite time to be processed. Often this processing time (or delay, denoted by τ) has been ignored and the systems modelled by ordinary differential equations (ODEs). But in many cases the observed behaviour does not match that predicted by ODEs. So we have to incorporate the delay into our models, using delay differential equations (DDEs), where the evolution of a dependent variable at time t depends on its value at time t − τ. Solving a DDE is a mathematically difficult task: even when the delay is small, the solution of a DDE can be completely different from the solution when the delay is zero.

In this module, I shall begin by motivating the use of DDEs with some simple examples. Then I will show you how to solve some types of DDE using methods from ODEs. But the core of the module is devoted to the D-subdivision method, which is the most widely used method in the field to determine the stability of solutions to DDEs. Throughout the module, I will show real world applications and provide you with many exercises to help you to develop your understanding of this important area.

Students should have a basic background in dynamics (for example, a familiarity with topics in Strogatz's book *Nonlinear Dynamics and Chaos* would be useful) and may be interested in applications.

I Lesson: Some books. Importance of initial conditions. Examples of delay differential equations (DDEs). Differences between DDEs and ODEs (ordinary differential equations) - oscillations and jumps in linear DDEs, infinite number of solutions, no solutions, dangers of using Taylor expansions. Method of steps - how some DDEs can be solved as if they were ODEs.

II Lesson: The D-subdivision method - a general method to analyse linear DDEs with constant delay. The Hayes equation - the simplest DDE with constant delay. Derivation of stability chart for trivial solution, including direction of eigenvalue crossing of stability boundaries. Demonstration of infinite number of eigenvalues of characteristic equation for linear DDE.

III lesson: The Hayes equations: existence of periodic solutions for certain parameter values. Delayed oscillator equation with constant delay. Derivation of stability chart in absence of damping. Double Hopf points. Stability chart in presence of damping.

IV Lesson: Delayed feedback controllers (PDA) with constant delay. Cushing's equation (for problems with distributed delay). Use of Laplace transforms to obtain characteristic equation. Numerical methods for DDEs. Examples of state dependent delay equations and partial differential equations with delay.

V Lesson - Assessment: In the last day of the course, PhD students interested in receiving credits for the course will be asked to carry out an assignment during the class.

ECTS Credits: 3

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