

## **Avviso di minicorso**

Il Prof. **Costantinos Siettos**, *School of Applied Mathematics and Physical Sciences, National Technical University of Athens, Greece*, terrà, presso l'aula E, terzo piano del Dipartimento di Matematica e Applicazioni "Renato Caccioppoli" dell'Università di Napoli Federico II nei giorni

25 gennaio (ore 9,30-11,30 e 15-17)

26 gennaio (ore 9,30-11,30)

1° febbraio (ore 9,30-11,30)

2 febbraio (ore 9,30-11,30 e 15-17)

un corso dal titolo

### **A Short Introduction to Bifurcation Theory**

Per chiarimenti e informazioni è possibile rivolgersi a Gerardo Toraldo ([toraldo@unina.it](mailto:toraldo@unina.it)) e Salvatore Cuomo ([salvatore.cuomo@unina.it](mailto:salvatore.cuomo@unina.it))

# **A Short Introduction to Bifurcation Theory**

## **Description and Outline**

Many systems in Applied Mathematics, Physical Sciences, Biology, Ecology, Economics and Engineering are characterized by nonlinear behaviour such as multistability, phase transitions, oscillatory spatio-temporal patterns and chaos. Thus the quest for the systematic analysis of such phenomena revolves around the theoretical and numerical analysis of the solutions in the parametric space of the corresponding mathematical models that usually appear in the form of Ordinary and/or Partial Differential Equations.

The outline of this short course in Bifurcation Theory reads as follows:

### **A. Introduction**

1. Examples from real-world systems.
2. Solutions, trajectories of dynamical systems
3. The concept of the stability of solutions. Lyapunov's definitions. The local approximation.
4. Finding the Solutions: Simple Simulation vs. Iterative numerical methods
5. Invariant manifolds. Stable, Unstable Central.
6. Definition of Bifurcating solutions

### **B. Bifurcations and Stability of dynamical systems in one dimension**

1. The Implicit Function Theorem
2. Double bifurcation points
3. Criteria of local conditioned stability
4. The factorization theorem
5. Exchange of stability in double points

### **C. Normal Forms of Bifurcations in 1D**

1. Saddle-node, Transcritical bifurcation, Pitchfork bifurcation. Examples
2. Imperfection Theory and the Break of Symmetry

### **D. Dynamics in two dimensions and stability**

1. Construction of phase diagrams
2. Stable and unstable manifolds
3. Classification of the behaviour of linear systems

### **E. Oscillating solutions/ Limit Cycles. Bifurcation of limit cycles from fixed points in two dimensions**

1. Andronov-Hopf bifurcation. Normal Forms.
2. Criteria for the appearance and exclusion of limit cycles. Gradient systems, Bendixson criterion. Examples.
3. Relaxation Dynamics. Van der Pol oscillator (Mechanics).
4. Examples. Lorenz equations (Fluid Dynamics). FitzHugh-Nagumo Equations (Neuroscience)
5. Numerical methods for the computation of Limit Cycles.
6. Stability of Limit Cycles and Poincare Maps.